كلية الحاسبات والمعلومات

الفرقة الثالثة تخلف من الفرقة الثانية

الفصل الدراسي الاول

2019م-2020م

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نموذج اجابة+صورة من الاسئلة

ورقة كاملة

المادة: رياضيات (3)

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أستاذ مساعد بقسم الرياضيات بكلية العلوم ببنها

(صورة من الاسئلة)

(تخلفات)

Benha University
Faculty of Computers & Informatics

Subject: Mathematics(3)
Time: 3 hours
Class: 2st Year
Students,term1 (December 2019)

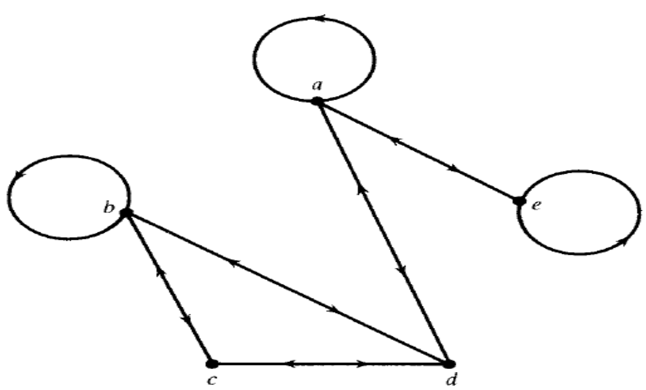
Answer the following questions:

Q1)

i) Let A = {1, 2, 3} and B = {1, 2}. Determine whether each of the following is **true** or **false** and give **a brief justification**. **[10 marks]**

(i)	$B \in p(A)$	(ii)	$B \in A$
(iii)	$A \in p(A)$	(iv)	$A \subseteq p(A)$
(v)	$B \subseteq p(A)$	(vi)	$\{\{1\}, B\} \subseteq p(A)$
(vii)	$\phi \in p(A)$	(viii)	$\phi \subseteq p(A)$

ii) Consider the directed graph given in the following figure of a relation R on the set A = {a, b, c, d, e}. **[10 marks]**



Give a brief justification for Determine whether the relation is

a. Reflexive;
b. Symmetric;
c. Anti-symmetric;
d. Transitive.

Q2)

i) Show that ($\bar{p} \vee \bar{q}$) and ($\overline{p \wedge q}$) are logically equivalent. **[10 marks]**

ii) Consider *S* = *p(A)*, the set of all subsets of a set *A*, together with the binary operation of intersection, ∩.

e. Is ∩ commutative on *S*? **[5 marks]**

f. What is the identity element? **[5 marks]**

g. Which elements, if any, have inverses? **[5 marks]**

Q3)

i) Given $f(x) = 3x^2 - x + 10$ and $g(x) = 1 - 20x$ find each of the following

(a) $(f \circ g)(x)$ **(b)** $(g \circ f)(x)$ **[6 marks]**

ii) Given $h(x) = \dfrac{x+4}{2x-5}$ find $h^{-1}(x)$ **[9 marks]**

Q4)

i) Find the inverse of the matrix $A = \begin{pmatrix} 6 & 2 \\ 4 & 1 \end{pmatrix}$. **[5 marks]**

ii) Find the eigenvalues and the eigenvectors of the matrix $A = \begin{pmatrix} 7 & 3 \\ 3 & -1 \end{pmatrix}$

[10 marks]

Q5)

i) By using the definition of the definite integral compute the following integral $\int_0^2 x^2 dx$

[10 marks]

ii) Use the principle of mathematical proof to prove that

$$1 + 3 + 5 + \ldots.. + (2n-1) = n^2$$ **[10 marks]**

GOOD LUCK,

Dr. Ahmed Megahed **Dr. Gamal Mosa**

نموذج الاجابة

اجابة السؤال الاول

i)

Solution

(i) True: B is a subset of A so B is an element of its power set.

(ii) False: B is a set but the elements of A are numbers, so B is not an element of A.

(iii) True: since $A \subseteq A$ it follows that $A \in \mathscr{P}(A)$. In fact, as noted above, this is the case for any set A.

(iv) False: the elements of A are numbers whereas the elements of $\mathscr{P}(A)$ are sets (namely subsets of A). Hence the elements of A cannot also be elements of $\mathscr{P}(A)$, so $A \nsubseteq \mathscr{P}(A)$.

(v) False: for the same reasons as given in part (iv).

(vi) True: $\{1\} \in \mathscr{P}(A)$ (since $\{1\} \subseteq A$) and $B \in \mathscr{P}(A)$ (part (i)) so each element of the set $\{\{1\}, B\}$ is also an element of $\mathscr{P}(A)$; hence $\{\{1\}, B\} \subseteq \mathscr{P}(A)$.

(vii) True: since $\varnothing \subseteq A$, we have $\varnothing \in \mathscr{P}(A)$.

(viii) True: $\varnothing \subseteq X$ for every set X and $\mathscr{P}(A)$ is certainly a set, so $\varnothing \subseteq \mathscr{P}(A)$.

ii)

From this diagram we can see that:

(i) R is not reflexive, since there is no arrow from *c* to itself, for example.

(ii) R is symmetric, since every arrow connecting distinct points is bidirectional.

(iii) R is not anti-symmetric, since, there are arrows from *a* to *d*, and from *b* to *a*, but a≠b.

(iv) R is not transitive since, for instance, there are arrows from *a* to *d*, and from *d* to *b*, but not from *a* to *b*.

اجابة السؤال الثانى

i)

Solution

We draw up the truth table for $\bar{p} \vee \bar{q}$ and also for $\overline{p \wedge q}$.

p	q	$\bar{p}$	$\bar{q}$	$\bar{p} \vee \bar{q}$	$p \wedge q$	$\overline{p \wedge q}$
T	T	F	F	F	T	F
T	F	F	T	T	F	T
F	T	T	F	T	F	T
F	F	T	T	T	F	T

ii)

a. Yes.
b. A.
c. A is the only element with an inverse. (A is self-inverse.)

اجابة السؤال الثالث

(a) $(f \circ g)(x) = f(g(x)) = 3g(x)^2 - g(x) + 10 = 3(1-20x)^2 - (1-20x) + 10$

(b) $(g \circ f)(x) = g(f(x)) = 1 - 20f(x) = 1 - 20(3x^2 - x + 10)$

ii) The first couple of steps are pretty much the same as the previous examples so here they are,

$$y = \frac{x+4}{2x-5} \Rightarrow x = \frac{y+4}{2y-5}$$

Now, be careful with the solution step. So, we have:

$$x(2y-5) = y+4 \Rightarrow y = \frac{4+5x}{2x-1}$$

$$\Rightarrow h^{-1}(x) = \frac{4+5x}{2x-1}$$

اجابة السؤال الرابع

i)

$$\Rightarrow |A| = -2, \quad A^T = \begin{pmatrix} 6 & 4 \\ 2 & 1 \end{pmatrix}, \quad adjA = \begin{pmatrix} 1 & -2 \\ -4 & 6 \end{pmatrix}$$

$$\Rightarrow A^{-1} = \begin{pmatrix} \frac{-1}{2} & 1 \\ 2 & -3 \end{pmatrix}$$

$$A - \lambda I = \begin{pmatrix} 7-\lambda & 3 \\ 3 & -1-\lambda \end{pmatrix}$$

ii) $\Rightarrow \det(A - \lambda I) = 0 \Rightarrow \begin{vmatrix} 7-\lambda & 3 \\ 3 & -1-\lambda \end{vmatrix} = 0$

$$\Rightarrow \lambda^2 - 6\lambda - 16 = 0 \Rightarrow \lambda = -2 \quad or \quad \lambda = 8$$

Then the matrix has two eigenvalues $\lambda = -2 \quad or \quad \lambda = 8$

Firstly, for $\lambda = 8$ we have:

$$A - \lambda I = \begin{pmatrix} -1 & 3 \\ 3 & -9 \end{pmatrix} = \widehat{B}$$

Now we must solve $\widehat{B}x = 0 \Rightarrow \begin{pmatrix} -1 & 3 \\ 3 & -9 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} -1 & 3 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow 3x_2 = x_1$$

If we take $x_2 = 1 \Rightarrow x = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

By the same way for $\lambda = -2$ we have $x = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$

Finally, the matrix $A = \begin{pmatrix} 7 & 3 \\ 3 & -1 \end{pmatrix}$ has two eigenvalues $\lambda = -2 \quad and \quad \lambda = 8$

Also it has two eigenvectors $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$.

اجابة السؤال الخامس

i) We know that $\int_a^b f(x)dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i)\Delta x$

Where $\Delta x = \frac{b-a}{n} = \frac{2}{n}$, $x_i = \frac{2i}{n}$, $f(x_i) = x_i^2 = \frac{4i^2}{n^2}$

$$\Rightarrow \int_0^2 x^2 dx = \lim_{n \to \infty} \sum_{i=1}^{n} (\frac{4i^2}{n^2})\frac{2}{n} = \lim_{n \to \infty} \sum_{i=1}^{n} (\frac{8i^2}{n^3})$$

$$= \lim_{n \to \infty} (\frac{8}{n^3} \sum_{i=1}^{n} i^2)$$

But $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$,

$$\Rightarrow \int_0^2 x^2 dx = \lim_{n \to \infty} (\frac{8}{n^3} (\frac{n(n+1)(2n+1)}{6})) = \frac{8}{3}$$

ii) At n=1, L. H. S. = R. H. S.=1

Let the relation is true for n=k

$$\Rightarrow 1+3+5+.....+(2k-1) = k^2 \qquad (1)$$

Now we try to prove that the relation is true for n=k+1

$$\Rightarrow L.H.S. = 1+3+5+.....+(2k-1)+(2k+1) = k^2 + 2k + 1 = (k+1)^2 = R.S.H$$

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