كلية الحاسبات والمعلومات

الفرقة الثانية

الفصل الدراسي الاول

2020-2019

تاريخ الامتحان: 2020/1/15

نموذج اجابة ورقة كاملة

المادة: رياضيات (3)

أستاذ المادة : د / أحمد مصطفي عبدالباقي مجاهد

استاذ مساعد بقسم الرياضيات بكلية العلوم ببنها

صورة من الاسئلة

Benha University
Faculty of Computers & Informatics

Subject: Mathematics(3)
Time: 3 hours
Class: 2st Year Students,
term1 (Jan 2020) Final Exam

Answer the following questions:

Q1)

a)- By using the principles of mathematical induction prove that $2^{2n}-1$ is divisible by 3. **[10 marks]**

b)- By using the inverse matrix, solve the following system. **[10 marks]**

$$2x+2y-6z=4$$
$$-x+y+2z=3$$
$$-3x+5y+3z=-1$$

Q2)

a)- By using the definition of the definite integrals, finds $\int_0^2 x^2 dx$ **[10 marks]**

b)- Find the eigenvalues and the eigenvectors of this matrix **[15 marks]**

$$\begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$$

Q3: **[15 marks]**

a) Let $\{A_i : i \in I\}$ be an indexing family of subsets of universal set U then show that for any non-empty subset B of U

$$\boldsymbol{B} \cap (\cup_{i\in I} \boldsymbol{A}_i) = \cup_{i\in I} (\boldsymbol{B} \cap \boldsymbol{A}_i)$$

b) Show that for all sets A, X, and Y

$$A \times (X \cap Y) = (A \times X) \cap (A \times Y).$$

Q4: **[15 marks]**

a) Using the Algebra of proposition to prove

$$\neg p \wedge \neg(p \wedge q) \equiv \neg p$$

b) Construct the truth table of the following compound propositions

$$(p \vee \neg q) \rightarrow q$$

Q5 **[20 marks]**

a) Let R be a relation defined on $\times N \; by\, (a,b)R(c,d) \Leftrightarrow ad = bc$. Prove that R is equivalence relation and find equivalence class of [2,5], and [1,1].

b) Let * be an associative binary operation on a set S which has an inverse. prove that the inverse is unique

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GOOD LUCK,
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نموذج الاجابة

السؤال الاول:

Clearly that at n=1, then the quantity $2^{2n}-1$ is equal to 3 which is divisible by 3.

Let the relation is true for n=k, which means that

$$2^{2k}-1 \text{ is divisible by } 3 \qquad (*)$$

Now we try to prove the relation when n=k+1

At n=k+1 , we have

$$2^{2k+2}-1=2^2 2^{2k}-1=(3+1)2^{2k}-1=3 2^{2k}+2^{2k}-1$$

Which is automatically divisible by 3.

b)

المعادلتين السابقتين يمكن كتابتها على الصورة

$$\begin{pmatrix} 2 & 2 & -6 \\ -1 & 1 & 2 \\ -3 & 5 & 3 \end{pmatrix} . \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 1 \end{bmatrix}$$

أو على الصورة

$$A \cdot X = C$$

ومنها نجد أن

$$A = \begin{pmatrix} 2 & 2 & -6 \\ -1 & 1 & 2 \\ -3 & 5 & 3 \end{pmatrix} \Rightarrow \therefore |A| = -8 \neq 0$$

$$\therefore A^{-1} = \begin{pmatrix} \frac{7}{8} & \frac{9}{2} & \frac{-5}{4} \\ \frac{3}{8} & \frac{3}{2} & \frac{-2}{8} \\ \frac{2}{8} & 2 & \frac{-4}{8} \end{pmatrix}$$

وعلى ذلك يكون $X = A^{-1} \cdot C$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{146}{8} \\ \frac{50}{8} \\ \frac{15}{2} \end{bmatrix}$$

ومن هاتين المعادلتان نجد أن $x = 146/8 \quad , \quad y = 50/8, \quad z = 15/2$

السؤال الثاني:

a) By using the definition of the definite integrals, finds $\int_0^2 x^2 dx$

From the definition of the definite integrals we have:

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

Where $\Delta x = \frac{b-a}{n}, \; x_i = a + i\Delta x$, then in our problem $a = 0, b = 2, f(x) = x^2$

$$\Rightarrow \Delta x = \frac{2}{n}, \; x_i = \frac{2i}{n} \Rightarrow f(x_i) = \frac{4i^2}{n^2}$$

$$\Rightarrow \int_0^2 x^2 dx = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{8i^2}{n^3} = \frac{8}{n^3} \lim_{n\to\infty} \sum_{i=1}^{n} i^2 = \frac{8}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) = \frac{8}{3}$$

b) Firstly we have to get the eigenvalues as follows:

$$\det(A - \lambda I) = 0 \Rightarrow \det \begin{pmatrix} 1-\lambda & -3 & 3 \\ 3 & -5-\lambda & 3 \\ 6 & -6 & 4-\lambda \end{pmatrix} = 0$$

$$\Rightarrow -\lambda^3 + 12\lambda + 16 = 0 \Rightarrow \lambda = 4, -2, -2$$

At $\lambda = 4$ we have

$$(A-\lambda I)=\begin{pmatrix}-3 & -3 & 3\\ 3 & -9 & 3\\ 6 & -6 & 0\end{pmatrix}\Rightarrow\begin{pmatrix}-3 & -3 & 3\\ 3 & -9 & 3\\ 6 & -6 & 0\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}=\begin{pmatrix}0\\ 0\\ 0\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}1 & 1 & -1\\ 0 & 1 & \frac{-1}{2}\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}=\begin{pmatrix}0\\ 0\\ 0\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}=x_3\begin{pmatrix}\frac{1}{2}\\ \frac{1}{2}\\ 1\end{pmatrix}$$

Then for any $x_3 \in \Re$, say $x_3 = 1$, the first eigenvalues is

$$\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}=\begin{pmatrix}\frac{1}{2}\\ \frac{1}{2}\\ 1\end{pmatrix}$$

At $\lambda = -2$ we have

$$(A-\lambda I)=\begin{pmatrix}3 & -3 & 3\\ 3 & -3 & 3\\ 6 & -6 & 6\end{pmatrix}\Rightarrow\begin{pmatrix}3 & -3 & 3\\ 3 & -3 & 3\\ 6 & -6 & 6\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}=\begin{pmatrix}0\\ 0\\ 0\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}1 & -1 & 1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}=\begin{pmatrix}0\\ 0\\ 0\end{pmatrix}$$

$$\Rightarrow\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}=x_1\begin{pmatrix}1\\ 1\\ 0\end{pmatrix}+x_3\begin{pmatrix}0\\ 1\\ 1\end{pmatrix}$$

Then for any $x_1, x_3 \in \Re$, say $x_1 = 1, x_3 = 1$, the second eigenvalues is

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

Q3)

[01] $x \in B \cap (\cup_{i\in I} A_i) \Leftrightarrow x \in B \wedge x \in \cup_{i\in I} A_i \Leftrightarrow x \in B \wedge \exists\, i_0 \in I: x \in A_{i_0} \Leftrightarrow$
$x \in (B \cap A_{i_0}) \Leftrightarrow x \in \cup_{i\in I} (B \cap A_i)$

[02] $:) \in A \times (X \cap Y) \Leftrightarrow a \in A \wedge x \in (X \cap Y) \Leftrightarrow (a,x) \in (A \times X) \wedge (a,x) \in (A \times Y)$
$\Leftrightarrow (a,x) \in (A \times X) \cap (A \times Y)$

Q4)

[01] $\neg p \wedge \neg(p \wedge q) \equiv \neg p \wedge (\neg p \vee \neg q) \equiv \neg p$ (absorption lows)

[02]

p	q	$\neg q$	$(p \vee \neg q)$	$(p \vee \neg q) \longrightarrow q$
T	*T*	*F*	*T*	*T*
T	*F*	*T*	*T*	*F*
F	*T*	*F*	*F*	*T*
F	*F*	*T*	*T*	*F*

Q5)

[01] **The relation R is Reflexive, symmetric, and transitive, hence R is equivalence**
[2,5]={(2,5), (4,10),(6,15), (8,20),…},
[1,1]={(1,1), (2,2), (3,3),…}

[02] **Suppose that $x \in S$ has inverses y and z then**

$y * x = x * y = e, \quad z * x = x * z = e$

Now

$y = y * e = y * (x * z) = (y * x) * z = e * z = z$

Hence the inverse of x is unique.

Dr. Ahmed Mostafa Megahed