Benha University	*Faculty of Computers & Informatics*
1st Term (January2017) Final Exam	*Date: 23/1/2017*
Class: 1st Year Students	*Time: 3 Hours*
Subject: Mathematics (1)	*Examiner: Dr. Amr Soleiman – Dr. Mohamed Nasr*

جامعة بنها – كلية الحاسبات والمعلومات

الفرقة: الاولى

يوم الامتحان: الاثنين

تاريخ الامتحان: ٢٣ / ١ / ٢٠١٧ م

المادة : رياضيات (١)

الممتحن: د . / عمرو سليمان محمود

استاذ مساعد بقسم الرياضيات بكلية العلوم

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الامتحان+ نموذج إجابة

ورقة كاملة

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Answer all the following questions:

(Q1) Choose the correct answer: **[25 mark]**

1) If $f(x)=x^3+3x^2+4x+5$ and $g(x)=5$, then $(g\circ f)(x)=$

(A) $5x^2+15x+25$ (B) $5x^3+15x^2+20x+25$ (C) 225 (D) 5

2) $\lim\limits_{h\to 0}\frac{1}{h}\ln\left(\frac{2+h}{2}\right)$ is

(A) e^2 (B) 1 (C) $\frac{1}{2}$ (D) does not exist

3) If $f(x)=x^5-1$, then $f^{-1}(x)=$

(A) $\sqrt[5]{x+1}$ (B) $\frac{1}{\sqrt[5]{x+1}}$ (C) $\sqrt[5]{x-1}$ (D) $\frac{1}{\sqrt[5]{x}+1}$

4) Let $y=\cos(\arctan x)$. What is the range of this function?

(A) $\left\{y\,\middle|\,-\frac{\pi}{2}<y<\frac{\pi}{2}\right\}$ (B) $\{y\,|\,0<y\le 1\}$ (C) $\{y\,|\,0\le y\le 1\}$ (D) $\{y\,|\,-1<y<1\}$

5) If $f'(x)=-f(x)$ and $f(1)=1$, then $f(x)=$

(A) $\frac{1}{2}e^{-2x+2}$ (B) e^{-x-1} (C) e^{1-x} (D) e^{-x}

6) If $\sin x=e^y,\ 0<x<\pi$, what is $\frac{dy}{dx}$ in terms of x?

(A) $\tan x$ (B) $-\cot x$ (C) $\cot x$ (D) $\csc x$

7) $\lim\limits_{h\to 0}\frac{\sin(x+h)-\sin x}{h}$ is

(A) 0 (B) 1 (C) $\sin x$ (D) $\cos x$

8) $\frac{d^{99}}{dx^{99}}(\sin x)=$

(A) $\sin x$ (B) $\cos x$ (C) $-\sin x$ (D) $-\cos x$

9) $\int\frac{1}{\sqrt{25-x^2}}dx=$

(A) $\arcsin\frac{x}{5}+c$ (B) $\arcsin x+c$ (C) $\frac{1}{5}\arcsin\frac{x}{5}+c$ (D) $2\sqrt{25-x^2}+c$

10) If $f(x)=x^2$ and $g(x)=\sqrt{x}$, then the domain of $f\circ g$ is

(A) $(-\infty,0)$ (B) $[0,\infty)$ (C) R (D) $R-\{0\}$

انظر الورقة الثانية

(Q2) **[30 mark]**

a) Prove that $\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right), \quad x \in (-\infty, \infty)$?

b) Apply L'Hospital's rule to evaluate the following limits:

1) $\lim\limits_{x \to 0} \dfrac{\log_5(1+x)}{x}$,

2) $\lim\limits_{x \to \frac{\pi}{2}} \left(\sec x - \tan x\right)$,

3) $\lim\limits_{x \to 0^+} \left(\sin x\right)^x$,

4) $\lim\limits_{x \to \infty} \dfrac{\ln x}{x}$.

(Q3) **[30 mark]**

a) Complete and prove if : $y = \tan^{-1} x \Rightarrow \dfrac{dy}{dx} = \cdots$

b) Find y' for each of the following functions:

1) $y = e^{\sin(x^3+1)} + 5^x$,

2) $y = \cos^{-1}(\tan x)$,

3) $y = x^{\ln x} + \tan^{-1} e^x$,

4) $x\,y = \tan \dfrac{y}{x}$.

(Q4) **[30 mark]**

a) Find the Maclaurin series for $f(x) = \ln(1+x)$.

b) Evaluate:

1) $\displaystyle\int \frac{x-3}{\sqrt{9x^2+4}}\,dx$,

2) $\displaystyle\int \frac{1-\cos(e^{-2x})}{e^{2x}}\,dx$,

3) $\displaystyle\int \frac{x^2}{\sqrt{16-x^6}}\,dx$,

4) $\displaystyle\int \cot x \,.\ln(\sin x)\,dx$.

GOOD LUCK,

Dr. Amr Soleiman – Dr. Mohamed Nasr

The answer

Answer Q1:

No.	Answer
1)	D
2)	C
3)	A
4)	B
5)	C
6)	C
7)	D
8)	D
9)	A
10)	B

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Answer Q2 :

a)

$$\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right), \qquad x \in (-\infty, \infty).$$

$$y = \sinh^{-1} x \quad \Rightarrow \quad x = \sinh y = \frac{1}{2}(e^y - e^{-y}) \quad \overset{\times 2e^y}{\Rightarrow}$$

$$e^{2y} - 2xe^y - 1 = 0 \quad \Rightarrow \quad e^y = x \pm \sqrt{x^2+1} \quad \overset{\because e^y > 0}{\Rightarrow}$$

$$y = \sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right), \qquad x \in (-\infty, \infty).$$

b)

(1) $$\lim_{x\to 0} \frac{\log_5(1+x)}{x} \overset{\frac{0}{0}}{=} \lim_{x\to 0} \frac{1}{(1+x)\ln 5} = \frac{1}{\ln 5},$$

(2) $$\lim_{x\to\frac{\pi}{2}} \left(\sec x - \tan x\right) \overset{\infty-\infty}{=} \lim_{x\to\frac{\pi}{2}} \left(\frac{1}{\cos x} - \frac{\sin x}{\cos x}\right) = \lim_{x\to\frac{\pi}{2}} \frac{1-\sin x}{\cos x} \overset{\frac{0}{0}}{}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{-\cos x}{-\sin x} = 0,$$

(3) $$y = \lim_{x\to 0^+} \left(\sin x\right)^x \; 0^0 \Rightarrow \ln y = \ln \lim_{x\to 0^+} \left(\sin x\right)^x = \lim_{x\to 0^+} x \ln\left(\sin x\right)$$

$$= \lim_{x\to 0^+} \frac{\ln\left(\sin x\right)}{\frac{1}{x}} \overset{\frac{-\infty}{\infty}}{} = \lim_{x\to 0^+} \frac{\cot x}{\frac{-1}{x^2}}$$

$$= \lim_{x\to 0^+} \frac{-x^2}{\tan x} \overset{\frac{0}{0}}{=} \lim_{x\to 0^+} \frac{-2x}{\sec^2 x} = 0$$

$$\Rightarrow \quad y = e^0 = 1.$$

(4) $$\lim_{x\to\infty} \frac{\ln x}{x} \overset{\frac{\infty}{\infty}}{=} \lim_{x\to\infty} \frac{1}{x} = 0$$

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Answer Q3 :

a)

$$y = \tan^{-1} x \quad \Rightarrow \quad x = \tan y \quad \Rightarrow \quad 1 = y' \sec^2 y$$

$$y' = \frac{1}{\sec^2 y} = \frac{1}{1+\tan^2 y} = \frac{1}{1+x^2}$$

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

b)

(1) $y = e^{\sin(x^3+1)} + 5^x \Rightarrow \quad y' = 3x^2 \cos(x^3+1) e^{\sin(x^3+1)} + 5^x \ln 5$

(2) $y = \cos^{-1}\left(\tan x\right) \Rightarrow \quad y' = \dfrac{-\sec^2 x}{\sqrt{1-\left(\tan x\right)^2}}$

(3) $y = x^{\ln x} + \tan^{-1} e^x = z + \tan^{-1} e^x \Rightarrow \quad y' = z' + \dfrac{e^x}{1+(e^x)^2}$

$$z = x^{\ln x} \Rightarrow \ln z = \ln x (\ln x) = (\ln x)^2 \Rightarrow$$

$$z' = \frac{2\ln x}{x} \qquad \therefore \quad y' = \frac{2\ln x}{x} + \frac{e^x}{1+(e^x)^2}$$

(4) $x\,y = \tan \dfrac{y}{x} \quad \Rightarrow \quad x\,y' + y = \left(\dfrac{x\,y' - y}{x^2}\right) \sec^2 \dfrac{y}{x} \quad \Rightarrow$

$$\left(x - \frac{1}{x}\sec^2 \frac{y}{x}\right) y' = -y\left(1 + \frac{1}{x^2}\right) \sec^2 \frac{y}{x}$$

$$\therefore \quad y' = \frac{-y\left(1+\frac{1}{x^2}\right)\sec^2 \frac{y}{x}}{\left(x - \frac{1}{x}\sec^2 \frac{y}{x}\right)}$$

Answer Q4 :

a)

$$f(x) = \ln(1+x) \qquad f(0) = 0,$$

$$f'(x) = \frac{1}{1+x} \qquad f'(0) = 1,$$

$$f''(x) = \frac{-1}{(1+x)^2} \qquad f''(0) = -1,$$

$$f'''(x) = \frac{2}{(1+x)^3} \qquad f'''(0) = 2,$$

$$f^{iv}(x) = \frac{-2\times 3}{(1+x)^4} \qquad f^{iv}(0) = -3!,$$

....

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{2x^3}{3!} - \frac{3!x^4}{4!} + \cdots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \cdots$$

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b)

1) $$\int \frac{x-3}{\sqrt{9x^2+4}}\,dx = \int \frac{x}{\sqrt{9x^2+4}}\,dx - \int \frac{3}{\sqrt{9x^2+4}}\,dx$$
$$= \frac{1}{18}\int \frac{18x}{\sqrt{9x^2+4}}\,dx - \int \frac{3}{\sqrt{(3x)^2+2^2}}\,dx$$
$$= \frac{1}{9}\sqrt{9x^2+4} - \sinh^{-1}\left(\frac{3x}{2}\right) + C$$

2) $$\int \frac{1-\cos(e^{-2x})}{e^{2x}}\,dx = \int (1-\cos(e^{-2x}))\, e^{-2x}\,dx$$
$$= \int e^{-2x}\,dx + \int -\cos(e^{-2x})\, e^{-2x}\,dx$$
$$= \frac{-1}{2}\int -2\, e^{-2x}\,dx + \frac{1}{2}\int -2\cos(e^{-2x})\, e^{-2x}\,dx$$
$$= \frac{-1}{2} e^{-2x} + \frac{1}{2}\sin(e^{-2x}) + C$$

3) $$\int \frac{x^2}{\sqrt{16-x^6}}\,dx = \int \frac{x^2}{\sqrt{4^2-(x^3)^2}}\,dx = \frac{1}{2}\int \frac{2x^2}{\sqrt{4^2-(x^3)^2}}\,dx$$
$$= \frac{1}{2}\sin^{-1}\left(\frac{x^3}{4}\right) + C$$

4) $$\int \cot x \,.\ln(\sin x)\,dx = \frac{1}{2}\left[\ln(\sin x)\right]^2 + C$$