كلية الحاسبات والمعلومات

الفرقة الثالثة تخلف من الفرقة الثانية

الفصل الدراسي الاول

2021-2020

تاريخ الامتحان: 28 / 2 /2021

نموذج اجابة+صورة من الاسئلة

ورقة كاملة

المادة: رياضيات (3)

أستاذ المادة : د / أحمد مصطفي عبدالباقي مجاهد

أستاذ مساعد بقسم الرياضيات بكلية العلوم ببنها

(صورة من الاسئلة)

(تخلفات)

Benha University
Faculty of Computers & Informatics

Subject: Mathematics(3)
Time: 3 hours
Class: 2st Year Students,term2 (Feb. 2021)
Final Exam

<u>Answer the following questions:</u>

Q1)

i) Given $f(x)=3x^2-x+10$ and $g(x)=1-20x$ find each of the following

(a) $(g\circ f)(x)$ **(b)** $(f\circ g)(x)$

ii) Given $h(x)=\dfrac{x+4}{2x-5}$ find $h^{-1}(x)$

Q2)

i) Find the inverse of the matrix $A=\begin{pmatrix}6 & 2\\ 4 & 1\end{pmatrix}$.

ii) Find the eigenvalues and the eigenvectors of the matrix $A=\begin{pmatrix}7 & 3\\ 3 & -1\end{pmatrix}$

Q3)

i) By using the definition of the definite integral compute the following integral $\int_0^2 x^2 dx$

ii) Use the principle of mathematical proof to prove that

$$1+3+5+.....+(2n-1)=n^2$$

Q4)

[i] Let $\{A_i : i \in I\}$ be an indixing family of subsets of universal set U then show that for any non-empty subset B of U

$B \cup (\cap_{i\in I} A_i) = \cap_{i\in I} (B \cup A_i)$

[ii] Show that for all sets A, X, and Y

.$A \times (X \cup Y) = (A \times X) \cup (A \times Y)$

Q5)

[i] Using the Algebra of proposition to prove

$\neg p \wedge \neg(p \wedge q) \equiv \neg p$

[ii] Construct the truth table of the following compound propositions

$(p \vee q) \longrightarrow \neg q$

Q6)

[i] Let R be a relation defined on $\times N$ $by\ (a,b)R(c,d) \Leftrightarrow a + d = b + c$. Prove that R is equivalence relation and find equivalence class of [2,5], and [1,1].

[ii] Let $*$ be an associative binary operation on a set S which has an inverse . prove that the inverse is unique

---

GOOD LUCK,

Dr. Ahmed Megahed **Dr. Mostafa Hassan**

نموذج الاجابة

اجابة السؤال الاول

(a) $(f\,o\,g)(x)=f(g(x))=3g(x)^2-g(x)+10=3(1-20x)^2-(1-20x)+10$

(b) $(g\,o\,f)(x)=g(f(x))=1-20f(x)=1-20(3x^2-x+10)$

ii) The first couple of steps are pretty much the same as the previous examples so here they are,

$$y=\frac{x+4}{2x-5}\Rightarrow x=\frac{y+4}{2y-5}$$

Now, be careful with the solution step. So, we have:

$$x(2y-5)=y+4\Rightarrow y=\frac{4+5x}{2x-1}$$

$$\Rightarrow h^{-1}(x)=\frac{4+5x}{2x-1}$$

اجابة السؤال الثانى

i)

$$\Rightarrow |A|=-2,\quad A^T=\begin{pmatrix}6 & 4\\ 2 & 1\end{pmatrix},\quad adjA=\begin{pmatrix}1 & -2\\ -4 & 6\end{pmatrix}$$

$$\Rightarrow A^{-1}=\begin{pmatrix}\frac{-1}{2} & 1\\ 2 & -3\end{pmatrix}$$

$$A-\lambda I=\begin{pmatrix}7-\lambda & 3\\ 3 & -1-\lambda\end{pmatrix}$$

ii) $\Rightarrow \det(A-\lambda I)=0\Rightarrow\begin{vmatrix}7-\lambda & 3\\ 3 & -1-\lambda\end{vmatrix}=0$

$$\Rightarrow \lambda^2-6\lambda-16=0\Rightarrow \lambda=-2\quad or\quad \lambda=8$$

Then the matrix has two eigenvalues $\lambda = -2 \quad or \quad \lambda = 8$

Firstly, for $\lambda = 8$ we have:

$$A - \lambda I = \begin{pmatrix} -1 & 3 \\ 3 & -9 \end{pmatrix} = \hat{B}$$

Now we must solve $\hat{B}x = 0 \Rightarrow \begin{pmatrix} -1 & 3 \\ 3 & -9 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} -1 & 3 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow 3x_2 = x_1$$

If we take $x_2 = 1 \Rightarrow x = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

By the same way for $\lambda = -2$ we have $x = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$

Finally, the matrix $A = \begin{pmatrix} 7 & 3 \\ 3 & -1 \end{pmatrix}$ has two eigenvalues $\lambda = -2 \quad and \quad \lambda = 8$

Also it has two eigenvectors $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$.

اجابة السؤال الثالث

i) We know that $\int_a^b f(x)dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i)\Delta x$

Where $\Delta x = \frac{b-a}{n} = \frac{2}{n}, \quad x_i = \frac{2i}{n}, \quad f(x_i) = x_i^2 = \frac{4i^2}{n^2}$

$$\Rightarrow \int_0^2 x^2 dx = \lim_{n \to \infty} \sum_{i=1}^{n} (\frac{4i^2}{n^2})\frac{2}{n} = \lim_{n \to \infty} \sum_{i=1}^{n} (\frac{8i^2}{n^3})$$

$$= \lim_{n \to \infty} (\frac{8}{n^3} \sum_{i=1}^{n} i^2)$$

But $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$,

$$\Rightarrow \int_0^2 x^2 dx = \lim_{n \to \infty} \left(\frac{8}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)\right) = \frac{8}{3}$$

ii) At n=1, L. H. S. = R. H. S.=1

Let the relation is true for n=k

$$\Rightarrow 1+3+5+.....+(2k-1)=k^2 \qquad (1)$$

Now we try to prove that the relation is true for n=k+1

$$\Rightarrow L.H.S.=1+3+5+.....+(2k-1)+(2k+1)=k^2+2k+1=(k+1)^2=R.S.H$$

اجابة السؤال الرابع

Question No. 4

$$x \in B \cup (\cap_{i\in I} A_i) \Leftrightarrow x \in B \vee x \in \cap_{i\in I} A_i \Leftrightarrow x \in B \vee \ \forall i \in I: x \in A_i \quad \Leftrightarrow$$
$$x \in (B \cup A_i \ ) \Leftrightarrow x \in \cap_{i\in I} (B \cup A_i)$$

$$(a,x) \in A \times (X \cup Y) \Leftrightarrow a \in A \wedge x \in (X \cup Y) \Leftrightarrow (a,x) \in (A \times X) \vee (a,x) \in (A \times Y)$$
$$\Leftrightarrow (a,x) \in (A \times X) \cup (A \times Y)$$

اجابة السؤال الخامس

Question No. 5

[01] $\neg p \wedge \neg(p \wedge q) \equiv \neg p \wedge (\neg p \vee \neg q) \equiv \neg p$ (absorption lows)

[02]

p	q	$\neg q$	$(p \vee q)$	$(p \vee q) \longrightarrow \neg q$
T	T	F	T	F
T	F	T	T	T
F	T	F	T	F
F	F	T	F	T

اجابة السؤال السادس

Question No. 6

[01] The relation R is Reflexive, symmetric, and transitive, hence R is equivalence
[2,5]={1,4), (2,5),(3,6), (4,7),…},
[1,1]={(1,1), (2,2), (3,3),…}

[02] Suppose that $x \in S$ has inverses y and z then
$y * x = x * y = e. \quad z * x = x * z = e$

Now

$y = y * e = y * (x * z) = (y * x) * z = e * z = z$

Hence the inverse of x is unique.

Dr. Ahmed Mostafa Megahed