كلية الحاسبات والذكاء الاصطناعي

المستوي الثاني

برنامج أمن المعلومات واكتشاف الادلة الجنائية الرقمية

برنامج تكنولوجيا الشبكات والمحمول

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نموذج اجابة   ورقة كاملة

المادة: معادلات تفاضلية

أستاذ المادة : أ.د / أحمد مصطفي عبدالباقي مجاهد

استاذ بقسم الرياضيات بكلية العلوم ببنها

صورة من الاسئلة

Faculty of Computers & Artificial Intelligence
2nd Term (June 2022) Final Exam
Networking and Mobile Technologies Program
Information Security and Digital Forensics Program
Course Code: NBS204 **Level:** 2th level
Subject: Differential Equations

Benha University
Date: 18 / 6 /2022
Time: 3 Hours
Total Marks: 50 Marks
Examiner(s): Dr. **Ahmed Megahed**

Answer the following questions [8 questions in 5 pages]:

Question No. 1 [6 Marks]

Solve the following homogenous ordinary differential equation

$$y' = \frac{y - x}{y + x}$$

Question No. 2 [5 Marks]

For the following problem, use an integrating factor to solve the given differential equation

$$(2x^2 + y)dx + (x^2 y - x)dy = 0$$

Question No. 3 [6 Marks]

Solve the following ordinary differential equation (Bernolli equation)

$$x^2 y' + 2xy = y^3$$

Question No. 4 [5 Marks]

Solve the following ordinary differential equation

$$y'' + 2y' + 2y = 0$$

Question No. 5 [6 Marks]

Solve the following third order ordinary differential equation

$$2y''' - 9y'' - 5y' = 0$$

Question No. 6 [5 Marks]

Solve the following non-homogenous ordinary differential equation

$$y''' - 5y'' + 5y' - 25y = e^{2x}$$

Question No. 7 [8 Marks]

Use the properties of binomial theorem to solve the following non-homogenous ordinary differential equation

$$y''-3y'+2y = x^2$$

Question No. 8 [9 Marks]

Solve the following non-homogenous ordinary differential equation

$$y''-5y'+6y = e^{4x}\cos 3x$$

GOOD LUCK,

Dr. Ahmed Megahed

Model Answer

Q1) To solve the following ODE,

$$y' = \frac{y-x}{y+x}$$

we observe that it is a homogenous ODE of degree one.

Let $\quad y = vx, \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{v-1}{v+1}$$

$$\Rightarrow \frac{dx}{x} + \frac{v+1}{v^2+1}dv = 0$$

$$\Rightarrow \ln x + \frac{1}{2}\ln(v^2+1) + \tan^{-1} v = C$$

$$\Rightarrow \ln x + \frac{1}{2}\ln((\frac{y}{x})^2+1) + \tan^{-1}\frac{y}{x} = C$$

Q2) Assume that we rewrite the following ODE

$$(2x^2 + y)dx + (x^2 y - x)dy = 0, \qquad (*)$$

In the following form $M\,dx + N\,dy = 0$

We observe that $\frac{\partial M}{\partial y} = 1, \quad \frac{\partial N}{\partial x} = 2xy - 1$

Let $\mu = e^{\int \frac{1}{N}(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x})dx}$

$$\Rightarrow \mu = e^{\int -\frac{2}{x}dx} = x^{-2}$$

Now, multiplying Eq. (*) by $\mu = x^{-2}$, we have

$$(2 + \frac{y}{x^2})dx + (y - \frac{1}{x})dy = 0, \qquad (**)$$

Clearly that (**) is an exact order ODE $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, then

The solution of (*) becomes as follows:

$$2x + \frac{1}{2}y^2 - \frac{y}{x} = C$$

Q3) We can rewrite the ODE in the following form

$$y' + \frac{2}{x}y = \frac{1}{x^2}y^3$$
$$\Rightarrow y^{-3}y' + \frac{2}{x}y^{-2} = \frac{1}{x^2}$$

Let $Z = y^{-2} \Rightarrow \frac{dZ}{dx} = -2y^{-3}\frac{dy}{dx}$

$\Rightarrow Z' - \frac{4}{x}Z = \frac{-2}{x^2}$ (This is a linear ODE)

Getting an integrating factor $\mu = e^{\int -\frac{4}{x}dx} = x^{-4}$

Then

$$\Rightarrow \frac{1}{x^4}Z = \int \frac{1}{x^4}(\frac{-2}{x^2})dx$$
$$\Rightarrow Z = \frac{2}{5x} + Cx^4$$
$$\Rightarrow y^{-2} = \frac{2}{5x} + Cx^4$$

Q4) Assume that the solution for the following ODE $y'' + 2y' + 2y = 0$ is in the following form:

$$y = e^{mx} \Rightarrow y' = me^{mx}, \; y'' = m^2e^{mx}$$

Then, after substitution, the original equation takes the form:

$$(m^2+2m+2)e^{mx}=0$$

This gives $m=-1\pm 2i$

Then the solution takes the form:

$$y=e^{-x}(c_1\cos 2x+c_2\sin 2x)$$

Q5) Assume that the solution for the following ODE $2y'''-9y''-5y'=0$ is in the following form:

$$y=e^{mx}\Rightarrow y'=me^{mx},\ \ y''=m^2e^{mx},\ \ y'''=m^3e^{mx}$$

Then, after substitution, the original equation takes the form:

$$(2m^3-9m^2-5m)e^{mx}=0$$

This gives $m_{1,2,3}=0,\ 5,\ -\frac{1}{2}$

Then the solution takes the form:

$$y=c_1+c_2e^{5x}+c_3e^{-0.5x}$$

Q6) Let $y=y_h+y_p$

To start obtaining y_h , we suppose that the equation is homogenous, i.e. it takes the form:

$$y'''-5y''+5y'-25y=0$$

Let

$$y=e^{Dx}\Rightarrow y'=De^{Dx},\ \ y''=D^2e^{Dx},\ \ y'''=D^3e^{Dx}$$

Then, after substitution, the original equation takes the form:

$$(D^3-5D^2+5D-25)e^{Dx}=0$$

This gives $D_{1,2,3}=5,\sqrt{5}\,i,\ -\sqrt{5}\,i$

Then the solution of y_h takes the form:

$$y_h=c_1e^{5x}+c_2\cos\sqrt{5}\,x+c_3\sin\sqrt{5}\,x$$

To get y_p , we assume that

$$(D^3 - 5D^2 + 5D - 25)y_p = e^{2x}$$

$$y_p = \frac{1}{D^3 - 5D^2 + 5D - 25} e^{2x}$$

$$y_p = \frac{-1}{27} e^{2x}$$

Then, the general solution for (*) becomes:

$$y = y_h + y_p = c_1 e^{5x} + c_2 \cos\sqrt{5}\,x + c_3 \sin\sqrt{5}\,x - \frac{1}{27} e^{2x}$$

Q7) Let $y = y_h + y_p$

To start obtaining y_h , we suppose that the equation is homogenous, i.e. it takes the form:

$$y'' - 3y' + 2y = 0$$

Let

$$y = e^{Dx} \Rightarrow y' = De^{Dx},\ \ y'' = D^2 e^{Dx}$$

Then, after substitution, the original equation takes the form:

$$(D^2 - 3D + 2)e^{Dx} = 0$$

This gives $D_{1,2} = 1,\ 2$

Then the solution of y_h takes the form:

$$y_h = c_1 e^x + c_2 e^{2x}$$

To get y_p , we assume that

$$(D^2 - 3D + 2)y_p = x^2$$

$$y_p = \frac{1}{D^2 - 3D + 2}x^2$$

$$y_p = \frac{1}{(D-1)(D-2)}x^2$$

$$= ((1-D)^{-1} - \frac{1}{2}(1-\frac{D}{2})^{-1})x^2$$

$$= ((1 + D + D^2 + \ldots) - \frac{1}{2}(1 + \frac{D}{2} + \frac{D^2}{4} + \ldots))x^2$$

$$= ((x^2 + 2x + 2) - \frac{1}{2}(x^2 + x + \frac{1}{2})) = \frac{1}{2}x^2 + \frac{3}{2}x + \frac{7}{4}$$

Then, the general solution for (*) becomes:

$$y = y_h + y_p = c_1 e^x + c_2 e^{2x} + \frac{1}{2}x^2 + \frac{3}{2}x + \frac{7}{4}$$

Q8) Let $y = y_h + y_p$

To start obtaining y_h , we suppose that the equation is homogenous, i.e. it takes the form:

$$y'' - 5y' + 6y = 0$$

Let

$$y = e^{Dx} \Rightarrow y' = De^{Dx}, \ \ y'' = D^2 e^{Dx}$$

Then, after substitution, the original equation takes the form:

$$(D^2 - 5D + 6)y_h = 0$$

This gives $D_{1,2} = 2, 3$

Then the solution of y_h takes the form:

$$y_h = c_1 e^{2x} + c_2 e^{3x}$$

To get y_p , we assume that

$$(D^2-5D+6)y_p = e^{4x}\cos 3x$$

$$y_p = \frac{1}{D^2-5D+6}e^{4x}\cos 3x$$

$$y_p = e^{4x}\frac{1}{(D+4-2)(D+4-3)}\cos 3x$$

$$= e^{4x}\frac{1}{3D-7}\cos 3x = e^{4x}\frac{3D+7}{(3D-7)(3D+7)}\cos 3x$$

$$= e^{4x}\frac{3D+7}{9D^2-49}\cos 3x = -e^{4x}\frac{3D+7}{130}\cos 3x$$

$$= \frac{-e^{4x}}{130}(-9\sin 3x + 7\cos 3x)$$

Then, the general solution for (*) becomes:

$$y = y_h + y_p = c_1e^{2x} + c_2e^{3x} + \frac{e^{4x}}{130}(9\sin 3x - 7\cos 3x)$$

Dr. Ahmed Mostafa Megahed